Optimal Energy Management for Hybrid Aircraft

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Outline

Background

- Aviation industry currently contributes around 2% of worldwide CO_2 emissions and has committed to reducing that level.
- Short-haul flights (< 3h) represent 53% of air traffic.

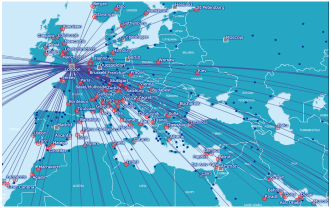


Figure 1: Short-haul flights from London (left) and typical short-haul airliner BAe 146 (right).

- Hybridisation of short-haul aircraft propulsion systems: aim is to reduce fuel consumption and hence pollutants.

Scenario

- BAe 146 equipped with a parallel hybrid-electric propulsion system (4 systems: 5MW gas turbine / 2MW electric motor).

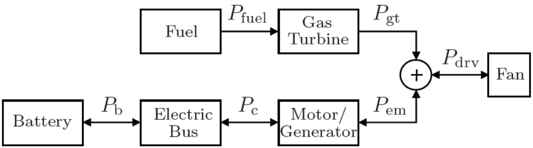


Figure 2: Parallel hybrid-electric propulsion.

Parameter	Symbol	Value	Units
Fuel mass	m_{fuel}	8000	kg
Battery SOC range	$[\underline{E}; \overline{E}]$	[221; 939]	MJ
Gas turbine power range	$[\underline{P}_{\mathrm{gt}}; \overline{P}_{\mathrm{gt}}]$	[0; 5]	MW
Motor power range	$[\underline{P}_{\mathrm{em}}; \overline{P}_{\mathrm{em}}]$	[0; 2]	MW
Number of systems	n	4	—

Table 1: Propulsion parameters.

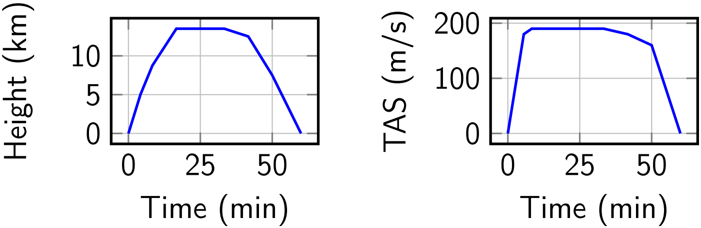
Scenario

Parameter	Symbol	Value	Units
Mass (MTOW)	m	42000	kg
Gravity acceleration	g	9.81	$\mathrm{m\,s^{-2}}$
Wing area	S	77.3	$\mathrm{m^2}$
Density of air	ρ	1.225	$\mathrm{kg\,m^{-3}}$
Lift coefficients	b_0	0.43	—
	b_1	0.11	$\mathrm{deg^{-1}}$
Drag coefficients	a_0	0.029	—
	a_1	0.004	$\mathrm{deg^{-1}}$
	a_2	$5.3\mathrm{e}{-4}$	$\mathrm{deg^{-2}}$
Flight time	T	3600	s

Table 2: Aircraft parameters.

Scenario

- A representative short-haul flight mission profile is prescribed:



Energy management problem

How to distribute the power demand across the different available sources of energy, while minimising fuel consumption ?

Naive solution

- A heuristic solution: charge depleting charge sustaining (CDCS) strategy, i.e. deplete the batteries fully at start.
- CDCS is commonly used in hybrid cars.

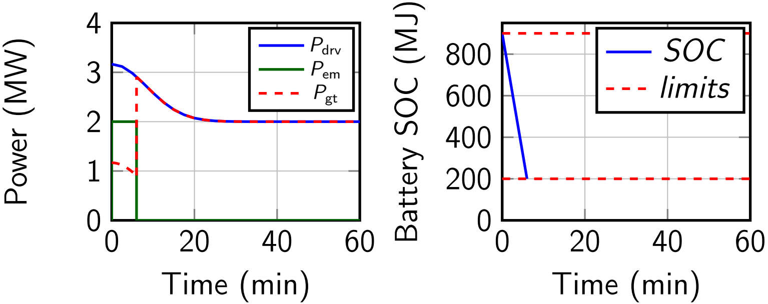


Figure 3: Energy mix with CDCS strategy (one couple gt / em).

- Can we do better ?

Problem formulation

- The energy management problem will be formulated as a convex program, solved using a model predictive control approach.
- The predicted performances are optimised subject to constraints on power flow and stored energy, and subject to the nonlinear aircraft dynamics, including nonlinear losses in powertrain components.

Modeling

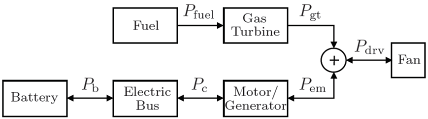
Goal

Minimise the cost function

$$J = -\int_0^T \dot{m}_{\mathsf{fuel}}(P_{\mathsf{gt}}, \omega_{\mathsf{gt}})\,\mathrm{d}t. \tag{1}$$

Assuming perfect mechanical coupling, the power balance is

$$P_{\mathsf{drv}}(t) = P_{\mathsf{gt}}(t) + P_{\mathsf{em}}(t). \tag{2}$$



- $P_{\mathsf{drv}} \equiv$ drive power.
- $P_{\mathsf{em}} \equiv$ electric power.
- $P_{\mathsf{gt}} \equiv$ gas turbine power.

Modeling: aircraft dynamics (P_{drv})

Assume a 2D point-mass model of an aircraft

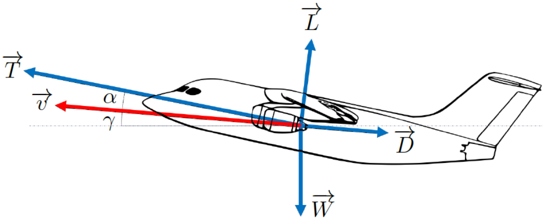


Figure 4: Aircraft forces and motion.

The equilibrium of forces yields

$$m\frac{\mathrm{d}}{\mathrm{d}t}(\vec{v}) = \vec{T} + \vec{L} + \vec{D} + \vec{W}.$$

Modeling: aircraft dynamics (P_{drv})

- Equations of motion:

$$m\frac{\mathrm{d}v}{\mathrm{d}t} + mg\sin\gamma = T\cos\alpha - \frac{1}{2}C_D(\alpha)\rho S v^2, \qquad (3)$$

$$mv\frac{\mathrm{d}\gamma}{\mathrm{d}t} + mg\cos\gamma = T\sin\alpha + \frac{1}{2}C_L(\alpha)\rho S v^2, \qquad (4)$$

where v and γ are the velocity and the flight path angle (prescribed), C_L and C_D the lift and drag coefficients, α the angle of attack.

Modeling: electric motor (P_{em})

- The electric bus is modelled as an equivalent circuit with internal resistance R and open-circuit voltage U such that

$$P_{\text{b}} = g\left(P_{\text{em}}(t), \omega_{\text{em}}(t)\right) = \frac{U^2}{2R}\left(1 - \sqrt{1 - \frac{4R}{U^2} P_{\text{c}}(P_{\text{em}}(t), \omega_{\text{em}}(t))}\right), \tag{5}$$

where ω_{em} is the electric motor shaft rotation speed.

- The electric losses in the electric motor are modelled as

$$P_{\text{c}} = h(P_{\text{em}}, \omega_{\text{em}}) = \kappa_2(\omega_{\text{em}})P_{\text{em}}^2 + \kappa_1(\omega_{\text{em}})P_{\text{em}} + \kappa_0(\omega_{\text{em}}). \tag{6}$$

Modeling: electric motor (P_{em})

- Battery state of charge (SOC) update equation:

$$\dot{E} = -g\big(P_{\text{em}}(t), \omega_{\text{em}}(t)\big). \tag{7}$$

- Constraints on the electric motor and battery limitations:

$$\underline{E} \leq E \leq \overline{E}, \tag{8}$$

$$\underline{P}_{\text{em}} \leq P_{\text{em}} \leq \overline{P}_{\text{em}}, \tag{9}$$

$$\underline{\omega}_{\text{em}} \leq \omega_{\text{em}} \leq \overline{\omega}_{\text{em}}. \tag{10}$$

Modeling: gas turbine (Pgt)

- Rate of change of the aircraft mass:

$$\dot{m} = \dot{m}_{\text{fuel}} = -f(P_{\text{gt}}(t), \omega_{\text{gt}}(t)), \qquad (11)$$

  where ω_{gt} is the gas turbine shaft rotation speed.

- Mechanical loss map:

$$f(P_{\text{gt}}, \omega_{\text{gt}}) = \beta_2(\omega_{\text{gt}})P_{\text{gt}}^2 + \beta_1(\omega_{\text{gt}})P_{\text{gt}} + \beta_0(\omega_{\text{gt}}). \qquad (12)$$

- Constraints on the gas turbine:

$$\underline{P}_{\text{gt}} \leq P_{\text{gt}} \leq \overline{P}_{\text{gt}}, \qquad (13)$$

$$\underline{\omega}_{\text{gt}} \leq \omega_{\text{gt}} \leq \overline{\omega}_{\text{gt}}. \qquad (14)$$

Optimisation problem

$$
\begin{aligned}
\min_{\substack{P_{\mathbf{gt}},\, P_{\mathbf{em}},\, P_{\mathbf{drv}} \\ m,\, E,\, \omega_{\mathbf{gt}},\, \omega_{\mathbf{em}}}} \quad & -\int_0^T \dot{m}(P_{\mathrm{gt}}, \omega_{\mathrm{gt}})\,\mathrm{d}t \qquad (15) \\
\text{s.t.} \quad & P_{\mathrm{drv}} = P_{\mathrm{gt}} + P_{\mathrm{em}}, \quad P_{\mathrm{drv}} = Tv\cos\alpha \\
& m\frac{\mathrm{d}v}{\mathrm{d}t} + mg\sin\gamma = T\cos\alpha - \frac{1}{2}C_D(\alpha)\rho S v^2 \\
& mv\frac{\mathrm{d}\gamma}{\mathrm{d}t} + mg\cos\gamma = T\sin\alpha + \frac{1}{2}C_L(\alpha)\rho S v^2 \\
& \dot{m} = -f(P_{\mathrm{gt}}, \omega_{\mathrm{gt}}) \\
& \dot{E} = -g(P_{\mathrm{em}}, \omega_{\mathrm{em}}) \\
& \underline{E} \le E \le \overline{E} \\
& \underline{P}_{\mathrm{gt}} \le P_{\mathrm{gt}} \le \overline{P}_{\mathrm{gt}} \\
& \underline{\omega}_{\mathrm{gt}} \le \omega_{\mathrm{gt}} \le \overline{\omega}_{\mathrm{gt}} \\
& \underline{P}_{\mathrm{em}} \le P_{\mathrm{em}} \le \overline{P}_{\mathrm{em}} \\
& \underline{\omega}_{\mathrm{em}} \le \omega_{\mathrm{em}} \le \overline{\omega}_{\mathrm{em}}
\end{aligned}
$$

Optimisation problem

$$\min_{\substack{P_{\mathbf{gt}},\, P_{\mathbf{em}},\, P_{\mathbf{drv}} \\ m,\, E,\, \omega_{\mathbf{gt}},\, \omega_{\mathbf{em}}}} \quad -\int_0^T \dot{m}(P_{\text{gt}}, \omega_{\text{gt}})\,\mathrm{d}t \tag{16}$$

$$\text{s.t.} \quad P_{\text{drv}} = P_{\text{gt}} + P_{\text{em}}, \quad P_{\text{drv}} = Tv\cos\alpha$$

$$m\frac{\mathrm{d}v}{\mathrm{d}t} + mg\sin\gamma = T\cos\alpha - \frac{1}{2}C_D(\alpha)\rho S v^2$$

$$mv\frac{\mathrm{d}\gamma}{\mathrm{d}t} + mg\cos\gamma = T\sin\alpha + \frac{1}{2}C_L(\alpha)\rho S v^2$$

$$\dot{m} = -f(P_{\text{gt}}, \omega_{\text{gt}})$$

$$\dot{E} = -g(P_{\text{em}}, \omega_{\text{em}})$$

$$\underline{E} \le E \le \overline{E}$$

$$\underline{P}_{\text{gt}} \le P_{\text{gt}} \le \overline{P}_{\text{gt}}$$

$$\underline{\omega}_{\text{gt}} \le \omega_{\text{gt}} \le \overline{\omega}_{\text{gt}}$$

$$\underline{P}_{\text{em}} \le P_{\text{em}} \le \overline{P}_{\text{em}}$$

$$\underline{\omega}_{\text{em}} \le \omega_{\text{em}} \le \overline{\omega}_{\text{em}}$$

Nonconvex !

Convex formulation

- Problem (16) is unfortunately non convex, making a real-time implementation computationally intractable.
- A convex formulation would ensure convergence to a global optimum if the problem is feasible.
- Moreover, convex programs can be solved efficiently.
- Convex programs are of the form

$$\begin{aligned} \min_{u,x} \quad & f_0(x,u) \\ \text{s. t.} \quad & f_i(x,u) = 0,\ i = 1,\dots,m \\ & g_j(x,u) \leq 0,\ j = 1,\dots,l \end{aligned}$$

with f_0 convex, f_i linear and g_j convex w.r.t. inputs and states.

Convex formulation

- The dynamical constraints (3) and (4) are nonlinear because of the thrust and aerodynamic terms:

$$m\frac{\mathrm{d}v}{\mathrm{d}t} + mg\sin\gamma = T\cos\alpha - \frac{1}{2}C_D(\alpha)\rho S v^2,$$

$$mv\frac{\mathrm{d}\gamma}{\mathrm{d}t} + mg\cos\gamma = T\sin\alpha + \frac{1}{2}C_L(\alpha)\rho S v^2.$$

- Thrust can be eliminated noting that $T\sin\alpha << L$ and $P_{\mathrm{drv}} = Tv\cos\alpha$. Hence

$$P_{\mathrm{drv}} = m\frac{\mathrm{d}}{\mathrm{d}t}(\frac{1}{2}v^2) + \frac{1}{2}C_D(\alpha)\rho S v^3 + mg\sin\gamma v, \tag{17}$$

$$mv\frac{\mathrm{d}\gamma}{\mathrm{d}t} + mg\cos\gamma = \frac{1}{2}C_L(\alpha)\rho S v^2. \tag{18}$$

Convex formulation

- Assuming

$$C_D(\alpha_i) = a_2\alpha_i^2 + a_1\alpha_i + a_0, \qquad a_2 > 0 \tag{19}$$
$$C_L(\alpha_i) = b_1\alpha_i + b_0, \qquad b_1 > 0 \tag{20}$$

and eliminating α from the EOM yields

$$P_{\text{drv}} = \eta_2 m^2 + \eta_1 m + \eta_0. \tag{21}$$

- η_2, η_1, η_0 are functions of the prescribed v and γ and $\eta_2 \geq 0$.
- From the objective (minimisation of the rate of change of aircraft mass), equality can be relaxed by inequality

$$P_{\text{gt}} + P_{\text{em}} = P_{\text{drv}} \geq \eta_2 m^2 + \eta_1 m + \eta_0. \tag{22}$$

Convex formulation

- The loss functions f and h from equations (12) and (6) are nonconvex because the coefficients depend on the rotation speed

$$f(P_{\text{gt}}, \omega_{\text{gt}}) = \beta_2(\omega_{\text{gt}})P_{\text{gt}}^2 + \beta_1(\omega_{\text{gt}})P_{\text{gt}} + \beta_0(\omega_{\text{gt}}),$$

$$h(P_{\text{em}}, \omega_{\text{em}}) = \kappa_2(\omega_{\text{em}})P_{\text{em}}^2 + \kappa_1(\omega_{\text{em}})P_{\text{em}} + \kappa_0(\omega_{\text{em}}).$$

- Assuming a gear ratio of 1:1 so that $\omega_{\text{drv}} = \omega_{\text{gt}} = \omega_{\text{em}}$, and noting that $\exists\Phi : \omega_{\text{drv}} = \Phi(v, h)$, the loss coefficients can be computed apriori from experimental data

$$f(P_{\text{gt}}) = \beta_2 P_{\text{gt}}^2 + \beta_1 P_{\text{gt}} + \beta_0, \quad (23)$$

$$h(P_{\text{em}}) = \kappa_2 P_{\text{em}}^2 + \kappa_1 P_{\text{em}} + \kappa_0. \quad (24)$$

- In practice, we can safely assume $\kappa_2 = 0$ and $\beta_2 = 0$.

Convex formulation

- The battery SOC update equation is nonlinear

$$\dot{E} = \underbrace{-\frac{U^2}{2R}\left(1 - \sqrt{1 - \frac{4R}{U^2}(\kappa_1 P_{\mathrm{em}} + \kappa_0)}\right)}_{\equiv g}.$$

- However, the change of variable

$$P_{\mathrm{b}} = g(P_{\mathrm{em}}),$$
$$P_{\mathrm{em}} = g^{-1}(P_{\mathrm{b}}) = -\frac{1}{\kappa_1}\left(\frac{R}{U^2}P_{\mathrm{b}}^2 - P_{\mathrm{b}} + \kappa_0\right),$$

where g^{-1} is concave ($\kappa_1 > 0$), yields a linear SOC constraint

$$\dot{E} = -P_b, \tag{25}$$

and a convex power balance

$$P_{\mathrm{gt}} \geq \eta_2 m^2 + \eta_1 m + \eta_0 - g^{-1}(P_{\mathrm{b}}). \tag{26}$$

Convex formulation

- To obtain a finite-dimensional optimisation problem, all equations are discretised with fixed sampling interval δ over a horizon N.

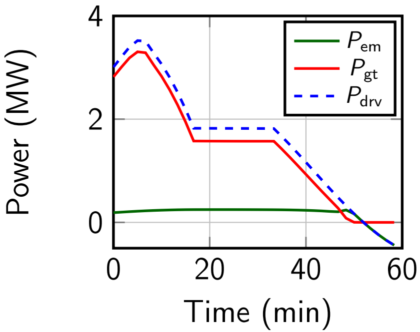
$$\min_{\substack{P_{\text{gt}}, P_{\text{b}}\, P_{\text{drv}} \\ m, E, \omega_{\text{gt}}, \omega_{\text{em}}}} \quad \sum_{i=0}^{N-1} f(P_{\text{gt},i})\delta \tag{27}$$

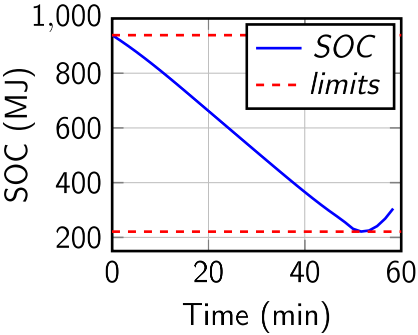
$$\begin{aligned}
\text{s.t.} \quad & P_{\text{gt},i} \geq \eta_{2,i} m_i^2 + \eta_{1,i} m_i + \eta_{0,i} - g_i^{-1}(P_{\text{b},i}) \\
& m_{i+1} = m_i - f_i(P_{\text{gt},i})\,\delta \\
& E_{i+1} = E_i - P_{\text{b},i}\,\delta \\
& \underline{E} \leq E_i \leq \overline{E} \\
& \underline{P}_{\text{gt}} \leq P_{\text{gt},i} \leq \overline{P}_{\text{gt}} \\
& \underline{P}_{\text{b},i} \leq P_{\text{b},i} \leq \overline{P}_{\text{b},i}
\end{aligned}$$

Results

- The convex program (27) is solved at each time step with a shrinking horizon.
- At each time step, the energy and mass are measured and the problem is updated with $m_0 = m(k\delta)$, $E_0 = E(k\delta)$.
- The optimisation problem was solved using CVX with SDPT3.

Results: Power mix

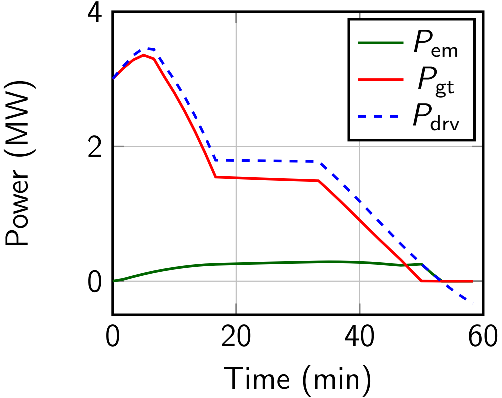




- The optimal strategy allows fuel savings of 3.2% over CDCS.
- Windmilling (i.e operation of the electric motor in generator mode to recharge the batteries) was allowed at the end.

Results: effect of increased fuel consumption

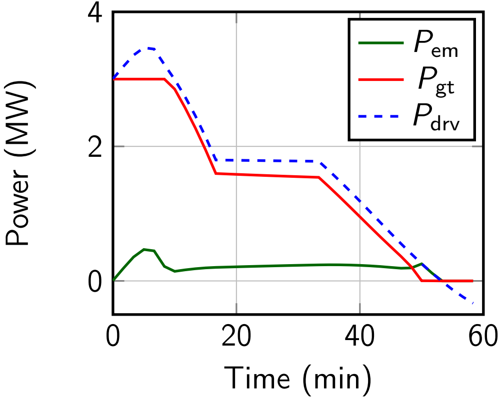
What if rate of fuel consumption is doubled ?



- The ratio gas turbine to electric power is higher at the start to reduce the mass (and hence P_{drv}) as fast as possible.

Results: effect of gas turbine saturation

What if maximum gas turbine power is limited to $\overline{P}_{\text{gt}} = 3MW$?



- The electric energy is mostly used at the power peak to compensate for gas turbine saturation.

Conclusion

- A convex optimisation program to solve the energy management problem of a (parallel) hybrid-electric aircraft.
- Model of the aircraft, electric motor and gas turbine was included.
- The problem was solved with CVX and the solution demonstrated significant energy savings w.r.t. heuristic strategies.
- Advantages: MPC-like strategy (feedback), convex program.
- Limitations: flight profile must be specified apriori.

Conclusion

Future work includes:

- The extension of the present algorithm to series-hybrid propulsion
- The development of 1st order solution methods exploiting the high degree of separability of the problem (ADMM)
- The implementation of a convex trajectory generation algorithm.

Thank you !