1P2H Digital Electronics 1

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Questions

1. Logic by switching circuits

The two switching circuits in Figure 1 are controlled by three switches A, B and C. Each switch has two positions (corresponding to logic 0 and 1), and in each circuit the switches are shown in their 0 position. Circuit (b) has a relay, which is shown in its activated position (i.e. extended when current flows through the coil). The lamp is not lit in both cases.

Treating the logical variable a to be true when switch A is in position 1, and similarly for switches B and C, write down a logical expression that is true when the lamp in circuit (a) is lit. Show that the lamp in circuit (b) is also lit when that same logical expression is true.

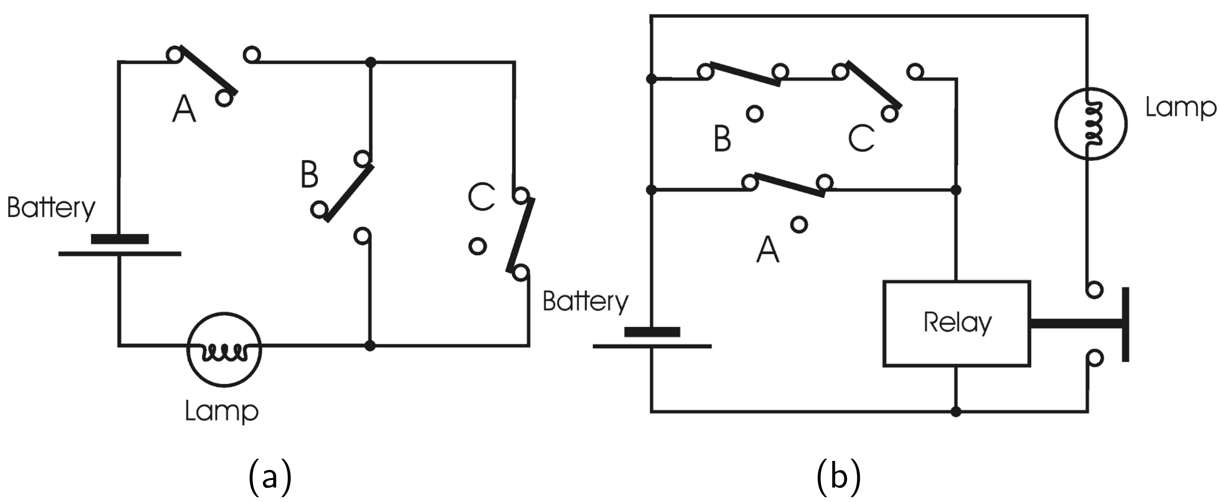

Figure 1: Switching circuits

2. Boolean algebra

(a). Construct the truth table for the logical function $f(a,b,c,d)$ where $f(a,b,c,d) = a.b.(a.\bar{c} + c.\bar{d}) + c.d + \bar{b}.\bar{d}$

(b). Write down equivalent expressions for f in sum-of-product (SOP) form and in canonical product-of-sums form.

(c). Demonstrate the following equivalence, where f is as defined in 2(a),

$$f(a,b,c,d) = (a + \bar{b} + c).(b + c + \bar{d}).(a + \bar{b} + d).$$

3. Karnaugh maps

Simplify the following logical formulae using Karnaugh maps (or otherwise):

(a). $f_1 = a.(b + \bar{c}) + b.\bar{c} + \bar{a}.\bar{c}$

(b). $f_2 = \bar{a}.b + \bar{b}.\bar{a}.\bar{d} + \bar{c} + \bar{a}.c.d$

(c). $f_3 = (a \oplus b).(c \oplus d) + a.b.c + \bar{a}.b.\bar{c}.\bar{d} + a.\bar{b}.c.d$

(d). $f_4 = \begin{cases} 1 & \text{if } a.b \text{ or } \bar{c}.d \\ 0 & \text{if } \bar{b}.\bar{d} \text{ or } \bar{a}.\bar{c}.\bar{d} \\ \text{don't care} & \text{otherwise} \end{cases}$

(e). What are the most complex Karnaugh maps with four variables, i.e. the ones that give the sum-of-product formulae with most terms?

4. MOSFET operation and CMOS

(a). Using the simple arguments given in the lecture notes explain how an n-channel MOSFET obtains a conducting channel between the drain and source contacts when $5\,\text{V}$ (logic 1) is applied to the gate electrode.

(b). What is the logical formula relating the output f to the inputs a, b and c in the CMOS circuit of Figure 2? (You may assume that the circuit is in standard form.)

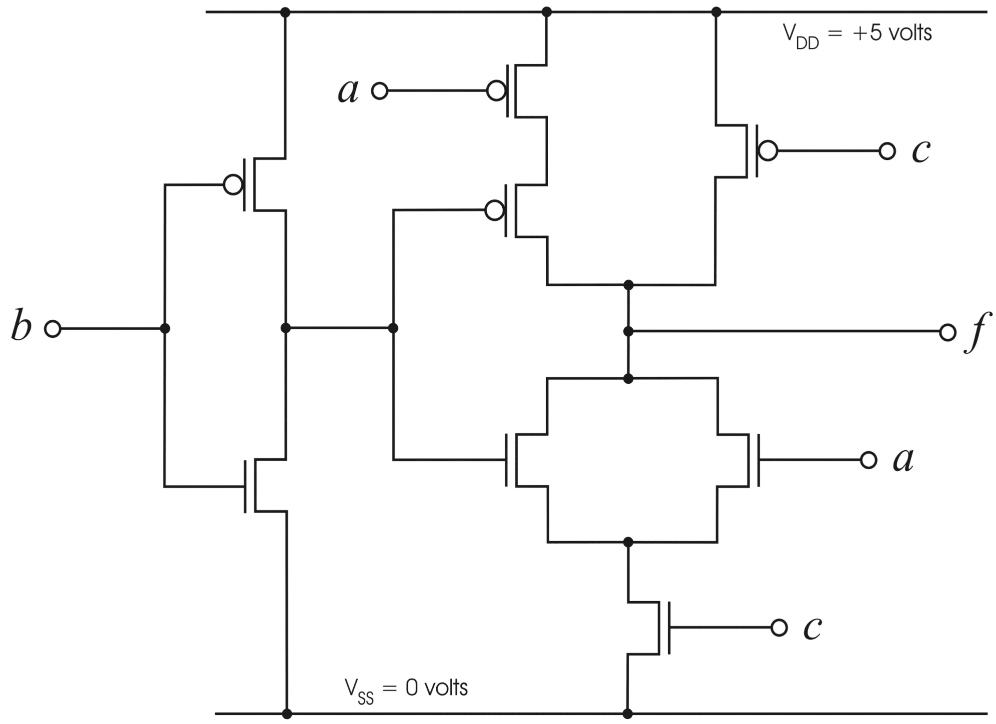

Figure 2: CMOS circuit

(c). Design a CMOS implementation of the functions $f = \overline{a.b}$, $g = a.b$.

5. Binary codes

(a). What is the largest decimal integer that can be represented in straight binary using 16 bits? What is the smallest?

(b). If (Hex) 8000 represents the integer (decimal) zero, what are the largest and smallest integers that can be represented using 16 bits now?

(c). What is the 2's complement for (decimal) -1?

(d). Draw two graphs on the same plot, the x-axis being straight binary from zero to the largest integer for 16 bits, the y-axis being the corresponding number represented by
(i) offset binary (ii) 2's complement.

(e). BCD and Gray codes are two common representations used in digital systems. Explain where and why they might be used and give an example of each.

(f). Gray code is often referred to as *reflected binary code*. Produce a table of binary values to illustrate why *reflected binary code* is a sensible description for Gray code and suggest an application.

6. Two's complement

This question helps you to understand how the use of two's complement representation of numbers enables the implementation of subtraction of integers using simple digital logic operations.

(a). How many 1s are there in the number $2^n - 1$?

(b). Subtract the binary number 01010101 from $2^8 - 1$. Use your result to explain how finding the one's complement of a number, i.e. replacing each 1 by a 0 and each 0 by a 1, followed by adding 1 to the result enables one to find the two's complement.

(c). Does adding 2^n to an n-bit number affect any of the least significant n bits of the resulting $(n+1)$-bit number?

(d). Use the above result and the fact that $a + (b - c) = (a - c) + b$ to explain how the two's complement representation of a negative number may be used to perform binary subtraction by using binary addition.

(e). Use two's complement arithmetic to subtract (Hex) 03 from (Hex) 40.

7. Adders/subtractors

(a). When is a Carry-out generated during addition?

(b). What is an Overflow and how is it different from a Carry-out?

(c). What is the disadvantage of a *ripple adder* and what is meant by *look ahead* carry?

8. Fixed point arithmetic

An 8-bit computer uses fixed-point arithmetic for positive numbers with four bits before the binary point, and four bits after the binary point.

(a). What is the smallest positive number representable in this form?

(b). What is the largest number representable?

(c). Is the addition of two such numbers straightforward to perform?

(d). Describe overflow and underflow in fixed-point multiplication. Perform the following multiplications and state whether there are any underflow or overflow problems associated with the desired operation:

(i). $0011.1100 \times 0010.0000$

(ii). $0000.0001 \times 0000.1000$

(iii). $0100.0000 \times 0100.0000$

9. Floating point number representation

A 16-bit floating point number consists of a sign bit, a 7-bit offset binary exponent, and an 8-bit mantissa with an implied 1. What is the largest number that can be represented if 126 is the largest number allowed in the exponent for normal numbers? Note that 127 is reserved for special purposes.

Some Answers

2. (b). $a.b + c.d + \bar{b}.\bar{d}$

   $(a{+}b{+}c{+}\bar{d}).(a{+}\bar{b}{+}c{+}d).(a{+}\bar{b}{+}c{+}\bar{d}).(a{+}\bar{b}{+}\bar{c}{+}d).(\bar{a}{+}b{+}c{+}\bar{d})$

3. $f_1 = a.b + \bar{c}$, $f_2 = \bar{a} + \bar{c} \equiv \overline{a.c}$, $f_3 = a.c + a.\bar{b}.d + \bar{a}.b.\bar{c} + \bar{a}.b.\bar{d}$
   or $f_3 = a.c + a.\bar{b}.d + \bar{a}.b.\bar{c} + b.c.\bar{d}$, $f_4 = a.b + d$

4. (b). $f_4 = \bar{a}.b + \bar{c}$

5. (a). 65535, 0 (b). 32767, -32768 (c). (Hex) FFFF

6. (a). n (b). 10101010 (c). No (e). (Hex) 3D

8. (a). $1/16$ (b). $255/16$
   (d). (i). 0111.1000 (ii). $000.0000(1)$ (underflow)
   (iii). $(1)0000.0000$ (overflow)

9. $0 : 111110 : 11111111 = 1 \times 2^{63} \times (2 - 1/256) \approx 1.841 \times 10^{19}$